Mathematical Statistics, II

MATH 322: End of Term Practice

The purpose of these end-of-term in-class activities is to give you practice in the many topics of this awesomely wonderful course. Hopefully, working on these things yourself will help to cement the material firmly in your brain, both for the test and for life beyond.

Estimated Time to Completion: 55 minutes

Question: What is the proportion of faculty who are in their offices during this period?

Steps:

a) Using your past experiences in this building, determine an appropriate prior distribution for that proportion. You may want to use the Secret Formulas® from last week to help you specify the parameters of the Beta distribution.

b) Start at the D-Wing stairwell, start walking (and counting) until you find the *sixth* professor in his/her office. Record the number of empty professor-offices in your notes. Each of you will start on a separate floor of this building.

c) Return to our island oasis and determine the posterior distribution of the proportion, given your individual data. Remember that you need to determine the likelihood function. So, what *is* the distribution of the random variable? Or, does it matter?

Questions to Answer:

1. Using the squared-error loss function, determine the Bayesian estimate for the proportion of professors in their offices during sixth period in SMC.
2. Using the absolute-error loss function, determine the Bayesian estimate for the proportion of professors in their offices during sixth period in SMC.
3. Using the 0/1-error loss function, determine the Bayesian estimate for the proportion of professors in their offices during sixth period in SMC.
4. Calculate an 80% credible interval for the proportion of professors in their offices during sixth period in SMC.
5. Test the hypothesis that the proportion of professors in their offices during sixth period in SMC is less than 5%.

6. Test the hypothesis that the proportion of professors in their offices during sixth period in SMC is greater than 1%.
7. Test the hypothesis that the proportion of professors in their offices during sixth period in SMC is between 1% and 3%.

Here are the Secret Formulas®

$$\alpha = \frac{\mu(1-\mu)}{\sigma^2}\mu - \mu$$

$$\beta = \frac{\mu(1-\mu)}{\sigma^2}(1-\mu) - (1-\mu)$$