STATISTICAL METHODS II
ASSIGNMENT 04
SOLUTIONS

This homework assignment deals with problems concerning comparing means of multiple groups. Please make sure you read the questions thoroughly and think about them *before* you begin your answer. The two questions use pseudo-data, data that I created from my computer. As always, you will need to use `R` to answer it. Download the two data sets from the web site (or link to them in your script). The filenames are given in the individual problems.

Your answers to the questions must be nicely typed. The answer should be at least paragraph in length and should follow the same pattern in what information is included:

- State the problem.
- State the null and alternative hypotheses in words.
- State the test you will use, its assumptions, and why you chose this test.
- In your answer, include the value of the test statistic, the degrees of freedom (if applicable), and the calculated p-value.
- Clearly draw the appropriate conclusion.

When you hand in this assignment, attach your `R` script to the back of the pages and include graphs immediately after (or with) the problem.

If you have any questions or issues, let me know as soon as possible.

Good luck!

Problem 04.1 ⟦4⟧

My client wishes to test which of three types of fertilizer are most effective in growing spelt wheat (*Triticum spelta*). To test this, he took a single plot of land and divided it into nine (9) sub-plots, planting spelt in each of the sub-plots. He then applied the first type of fertilizer to the three sub-plots nearest to the road; the second type to the next three nearest; the third type to the three farthest from the road. He then measured the height of 12 wheat plants in each plot prior to harvesting the wheat. A summary of the data is provided in the table below (and the entire data is provided in the datafile `wheat2.csv`).

Plot	Fertilizer	Mean Height
A1	N18-P51-K20	35.08
A2	N18-P51-K20	35.00
A3	N18-P51-K20	33.75
B1	N13-P00-K44	30.00
B2	N13-P00-K44	30.50
B3	N13-P00-K44	30.67
C1	N18-P18-K18	20.00
C2	N18-P18-K18	19.25
C3	N18-P18-K18	21.58

Is there a statistically significant difference in the height of the wheat plants based on the fertilizer used?

⋆⋆⋆

In addition to answering the above research question appropriately, please write a paragraph explaining why this experiment may be a bad experiment and that your results may be incorrect.

Solution:

- The dataset consists of measurements on wheat heights as affected by three different fertilizers. I want to determine if the fertilizers affect the wheat growth differently.
- The null hypothesis is that the fertilizers do not affect the wheat heights differently; that is, the average wheat heights are statistically equal across the three fertilizers. A glance at the boxplot (Figure 1) suggests that the means are different.
- I would prefer to use an Analysis of Variance procedure, as it is more powerful than a non-parametric test if the assumptions are met. This procedure assumes that the heights have the same variance across groups and that the heights are all distributed Normally in each group. The data does support the contention that the heights are distributed Normally within each group. However, the Bartlett test ($K^2 = 7.4446, df = 2, p = 0.024$) suggests that the heights have different variances across the three fertilizers. The Fligner test, on the other hand, does not find the differences statistically significant ($X^2 = 3.6911, df = 2, p = 0.158$). As the results of the two tests contradict one another, I will perform both the parametric Analysis of Variance test and the non-parametric Kruskal-Wallis test.
- Both the Analysis of Variance test ($F = 167.77, df_1 = 2, df_2 = 105, p \ll 0.0001$) and the Kruskal-Wallis test ($X^2 = 79.0801, df = 2, p \ll 0.0001$) reject the null hypothesis of equality of means.
- Because these tests agree, we conclude that there is an effect due to fertilizer on the heights of the wheat.

Now, it behooves us to examine the experiment. We do this because these numbers we have just analyzed come with meaning. The experiment was poorly (and fatally) designed. The plots received a specific type of fertilizer based on the plot's distance from the road. Thus, there is no way of extracting the effect of the road from the effect of the fertilizer. Fixing this error is impossible without further experiment.

This shows the importance of design in experiments. Had the researcher used each of the three fertilizers in each of the three strata (distance from the road), we would be able to control for the effect of the road in our analysis. ◇

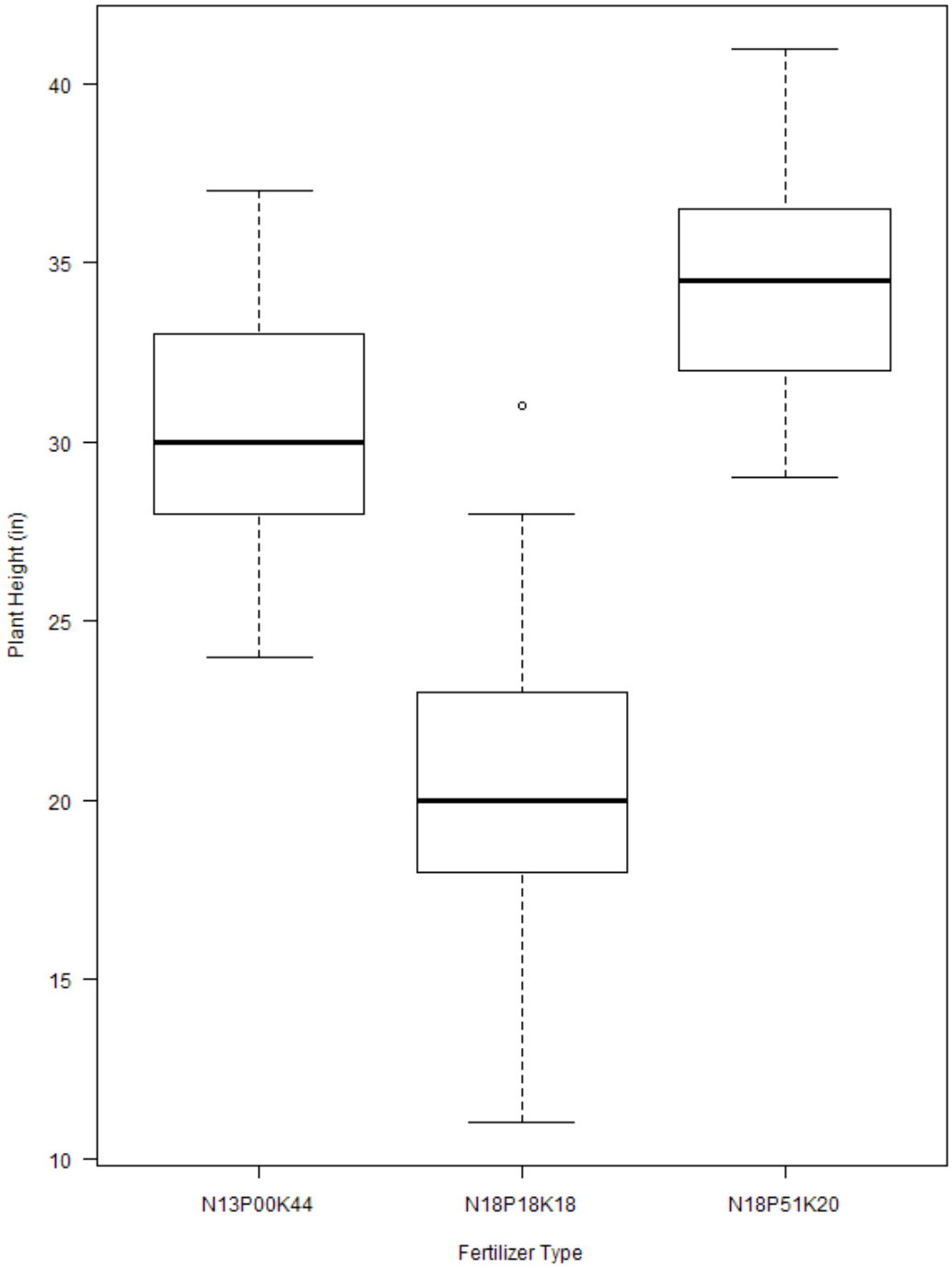

Figure 1. *Boxplot of the* `wheat2` *data.*

Problem 04.2 ⟦6⟧

Biomes are categories of similar climactic conditions on earth. Such conditions include plants types, soil types, and weather types. There are multiple biome classification schemes, each of which emphasizes certain aspects of the environment. The `biome2.csv` dataset uses the WWF system.

The dataset contains 30 sites around the United States. At each site, the biome type is determined, the distance to Stillwater is measured (in miles), the elevation of the site is measured (in feet above mean sea level), the mean fire return interval (mfri) is determined (in years) using tree ring data, and the standard deviation of the fire return interval (sfri) is calculated (in years).

Researchers hypothesize that the mean fire return interval is dependent upon the biome type. Does the sample data support this hypothesis?

⋆⋆⋆

In addition to answering the above question appropriately, answer the following additional questions completely:

(1) What is the variance of the mean fire return interval without taking the biome into consideration?
(2) What is the variance of the mean fire return interval taking the biome into consideration (you will have to add the variances in each biome)?
(3) Does knowing the biome give us statistically significant information about the expected mean fire return interval?

Solution:

- The dataset consists of mean fire return intervals (mfri) in several sites around the United States. In addition to the mfri, the biome type is recorded. The researcher wishes to know if the time between fires (mfri) varies among the five different biomes.
- The null hypothesis is that the mean fire return intervals do not vary across the biomes; that is, knowledge of the biome does not increase your knowledge of the mfri.
- I would prefer to use an Analysis of Variance procedure, as it is more powerful than a non-parametric test if the assumptions are met. This procedure assumes that the heights have the same variance across groups and that the heights are all distributed Normally in each group. The data does support the contention that the heights are distributed Normally within each group. However, the Bartlett test ($K^2 = 70.4287, df = 4, p \ll 0.0001$) and the Fligner test ($X^2 = 16.9955, df = 4, p \ll 0.0001$) both suggest that the mean fire return intervals have different variances across the three fertilizers. The boxplot of the data supports this conclusion (Figure 2).
- The Analysis of Variance test cannot be used due to the violation of equal variances. The Kruskal-Wallis test ($X^2 = 25.0651, df = 4, p \ll 0.0001$) indicates that we should reject the null hypothesis of equality of means.
- Due to the low p-value, we must conclude that there is a significant difference in mean fire return intervals across the five biomes.

For the additional information:

- The variance in the mean fire return interval is 41488.74.
- If we just add the variances together, we get 97067.15.
- Yes, as our Kruskal-Wallis test was statistically significant, we can conclude that information about the biome gives information about the mean fire return interval. In other words, biome and mfri are not independent.

◇

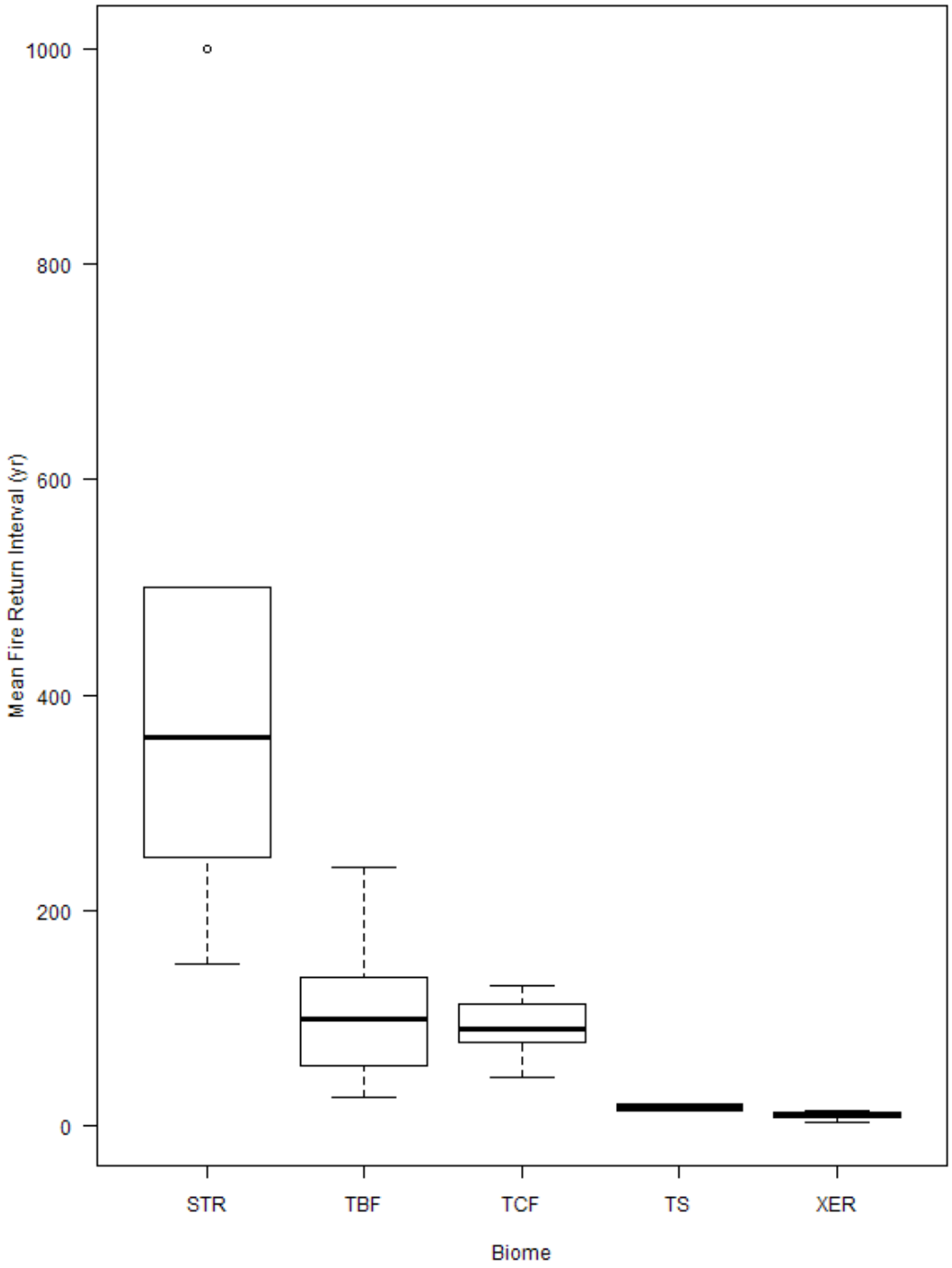

Figure 2. *Boxplot of the* `biome2` *data.*