STATISTICAL METHODS II
SOLUTIONS 01

Problem 01.1 [[5]]

Using both the tables at the back of the book and the R statistical environment, calculate the following probabilities. Of course, you will have to estimate the actual probabilities using the tables, but come close. Remember, you will need to attach your R script to the end of this homework assignment. Make sure you appropriately use comments in your R script.

In each of these problems, let $Z \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$, $T \sim t(dF = 4)$, $X \sim \mathcal{N}(\mu = 5, \sigma^2 = 3)$, and $R \sim \chi^2(dF = 6)$.

(1) $\mathbb{P}[Z > 1]$
(2) $\mathbb{P}[|Z| > 1]$
(3) $\mathbb{P}[|T| < 1]$
(4) $\mathbb{P}[|X + 1| < 6]$
(5) $\mathbb{P}[3 < R < 5]$

Solution: For these, we need to recall that the tables in the book (or with R) only give $\mathbb{P}[X \leq x]$. So, we have to use basic algebra to transform what we are given into an expression of this form (or a sum of expressions of this form).

(1) $\mathbb{P}[Z > 1]$

$$\begin{aligned} &= 1 - \mathbb{P}[Z \leq 1] \\ &= 1 - 0.8433 \\ &= 0.1587 \end{aligned}$$

In R: `1-pnorm(1)`

(2) $\mathbb{P}[|Z| > 1]$

$$\begin{aligned} &= \mathbb{P}[Z > 1] + \mathbb{P}[-Z > 1] \\ &= \mathbb{P}[Z > 1] + \mathbb{P}[Z \leq -1] \\ &= 1 - \mathbb{P}[Z \leq 1] + \mathbb{P}[Z \leq -1] \\ &= 1 - 0.8433 + 0.1567 \\ &= 0.3173 \end{aligned}$$

In R: `1-pnorm(1)+ pnorm(-1)`

(3) $\mathbb{P}\,[|T| < 1]$

$$
\begin{aligned}
&= 1 - \mathbb{P}\,[|T| > 1] \\
&= 1 - (\mathbb{P}\,[T > 1] + \mathbb{P}\,[-T > 1]) \\
&= 1 - (\mathbb{P}\,[T > 1] + \mathbb{P}\,[T \leq -1]) \\
&= 1 - (1 - \mathbb{P}\,[T \leq 1] + \mathbb{P}\,[T \leq -1]) \\
&= 1 - (0.1870 + 0.1870) \\
&= 0.6261
\end{aligned}
$$

In R: `1 - (1-pt(1,df=4) + pt(-1,df=4))`

(4) $\mathbb{P}\,[|X + 1| < 6]$

$$
\begin{aligned}
&= 1 - \mathbb{P}\,[|X + 1| > 6] \\
&= 1 - (\mathbb{P}\,[X + 1 > 6] + \mathbb{P}\,[-(X + 1) > 6]) \\
&= 1 - (\mathbb{P}\,[X > 5] + \mathbb{P}\,[X \leq -7]) \\
&= 1 - (1 - \mathbb{P}\,[X \leq 5] + \mathbb{P}\,[X \leq -7]) \\
&= 1 - (0.5000 + 3.17 \times 10^{-5}) \\
&= 0.5000
\end{aligned}
$$

In R: `1 - (1-pnorm(5, m=5,s=3) + pnorm(-7, m=5,s=3) )`

(5) $\mathbb{P}\,[3 < R < 5]$

$$
\begin{aligned}
&= \mathbb{P}\,[R < 5] - \mathbb{P}\,[R < 3] \\
&= 0.4562 - 0.1912 \\
&= 0.2650
\end{aligned}
$$

In R: `pchisq(5,df=6) - pchisq(3,df=6)`

◇

Problem 01.2 [[2]]

Calculate the power of an appropriate test of means, with $\alpha = 0.05$, and with the competing hypotheses being

$$H_1 : X \sim \mathcal{N}(\mu = 2, \sigma^2 = 1)$$
$$H_2 : X \sim \mathcal{N}(\mu = 4, \sigma^2 = 1)$$

Solution: As there is no requirement that we do this by hand, and as you will be doing it on the computer in the real world, let us use the computer (and the script from class) to ease our calculations.

First, remember the function we wrote:

```
powercalc <- function(m0,mA,s0,sA,alpha) {
    cv <- qnorm(1-alpha, mean=m0,sd=s0)
    power <- 1-pnorm(cv, mean=mA,sd=sA)
    return(power)
}
```

Now, we merely call this function using the information from the problem:

```
powercalc(2,4,1,1,0.05)
```

This shows that the power of this test is 0.63876. This also means that the probability of making a Type II error is 0.36124.

If we wanted to do this by hand, we would calculate the upper critical value for the null hypothesis ($cv = 3.645$) and then calculate the probability of $\mathbb{P}\left[X > 3.645\right]$ assuming the null was correct ($p = 0.36124$). ◇

Problem 01.3 [[3]]

Calculate a power curve using R and the script we went through in class on Thursday. The script (with many annotations) is available online on the "Topics" link.

The null hypothesis is

$$H_0 : X \sim \mathcal{N}(\mu = 0, \sigma = 4)$$

The alternative hypothesis is

$$H_A : X \sim \mathcal{N}(\mu > 0, \sigma = 4)$$

As usual, $\alpha = 0.05$.

Notice that the null hypothesis is a simple hypothesis, but the alternative is a compound hypothesis. Modify Part II (in that script) to calculate the power curve for these hypotheses. Note that you will have to use the second version (changing mean) and change the parameters to

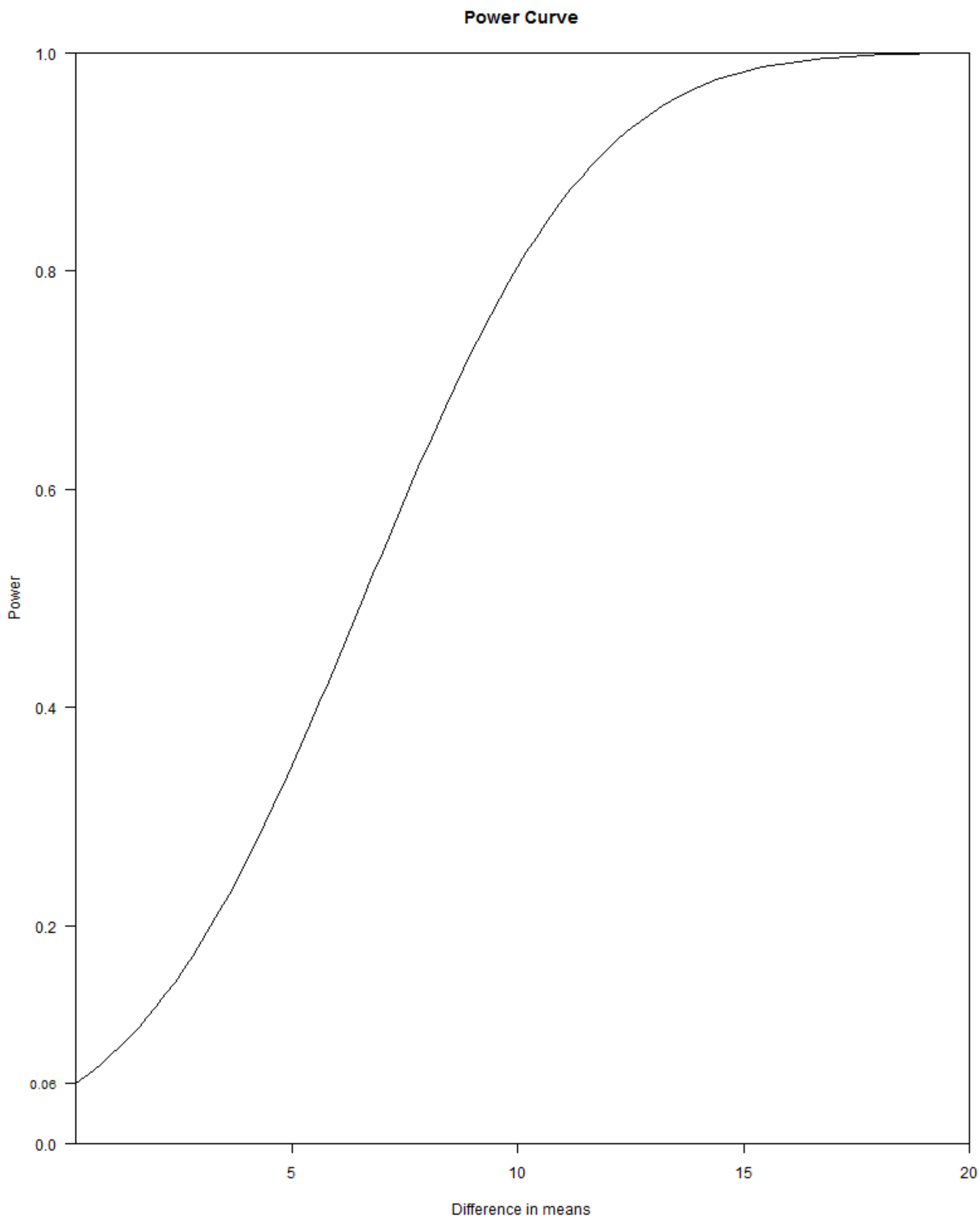
```
m0 <- 0; s0 <- 4; mA <- 0; sA <- 4; alpha <- 0.05
```

and the mean calculation to

```
mA <- i/5
```

When you have successfully run this script, please save the graph produced by right-clicking on it and saving it in your assignments folder (hopefully in a subfolder called "assignment01"). Select the 'Save as metafile. . . ' option.

Solution: There is not much to do (other than look at the R script and show the results). The R script is linked on the course webpage ('Assignments' link). The graph is on the next page. ◇


Power Curve
1.0
0.8
0.6
0.4
0.2
0.06
0.0
Power
5
10
15
20
Difference in means