0827 Prob & Counting

Note Title 8/27/2010

Today

1. Bivariate graphing
2. Probability
3. Counting

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1. Bivariate plotting

purpose: To help you determine if there is a relationship between two variables

given: an entity

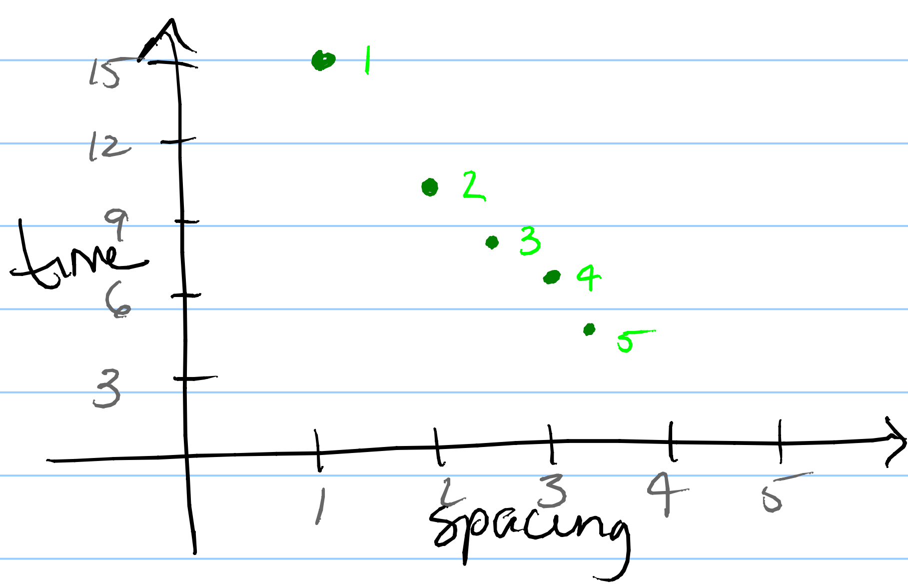
e.g.
- human
- country
- wheel
- gear
- computer program

and two measures on that entity

e.g.
- height & SAT score
- population & GDP per capita
- weight & radius
- teeth spacing & survival time
- structure style & execution time

for example

gear	teeth spacing	survival
1	1.0	15
2	2.0	10
3	2.5	8
4	3.0	7
5	3.3	5



2 Probability

population & universe
= all <u>possible</u> outcomes

event = a possible outcome

e.g. roll a die, get an odd number
$U = \{1, 2, 3, 4, 5, 6\}$
$A = \{1, 3, 5\}$

<u>Definition</u> of <u>Probability</u>

frequency $\#(A)$
relative frequency $\#(A)/N$

probability

$$P(A) = \lim_{N \to \infty} \#(A)/N$$

As U, A, etc. are just sets

Thus we can use set fctns to help calculate probabilities

Let $A = \{e_i \mid e_i \in A\}$
$B = \{e_i \mid e_i \in B\}$

e_i is an outcome

This means A is the set of all events/outcomes in A, and, likewise, B.

Then the following are true:

A OR B $\quad A \cup B := \{e_i \mid e_i \in A \text{ or } e_i \in B\}$

A AND B $\quad A \cap B = AB := \{e_i \mid e_i \in A \text{ and } e_i \in B\}$

NOT A $\quad A^c := \{e_i \mid e_i \notin A\}$

$A - B := \{e_i \mid e_i \in A \text{ and } e_i \notin B\}$

A XOR B $\quad A \Delta B = (A-B) \cup (B-A)$

examples

Make group of 2 from $\{M, L, C\}$

$A := \{M \text{ is in group}\}$

$B := \{L \text{ is in group}\}$

$U = \{ML, MC, LC\}$

$A = \{ML, MC\}$

$B = \{MC, LC\}$

$\Longrightarrow$

$$A \cup B = \{ML, MC, LC\}$$

$$AB = \{MC\}$$

$$A^c = \{LC\}$$

$$A - B = \{ML\}$$

$$B - A = \{LC\}$$

$$A \triangle B = \{ML, LC\}$$

de Morgan's Laws

$$A^c \cup B^c = \{LC, ML\} = (A \cap B)^c$$

$$A^c \cap B^c = \{\} = \emptyset = (A \cup B)^c$$

3 Counting

Fundamental Principle of Counting (FPC)

eg License Plates in Oklahoma
How many possible license plates?
plates consist of 6 figures:
three digits followed by three letters

0–9	0–9	0–9	A–Z	A–Z	A–Z
10	10	10	26	26	26

Solution:

by FPC, the total number of plates is:

$$10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 =$$

$$17{,}576{,}000$$

permutations

ex 15 players in a t-ball team
9 specific positions
How many ways of staffing those positions?

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15 14 13 12 11 10 9 8 7

$$= \frac{15!}{6!} = \frac{n!}{(n-r)!} = {}_nP_r$$

combinations

ex 15 players in a dodgeball team
9 on the court at a time
how many groups can be on the court?

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15 14 13 12 11 10 9 8 7

BUT those 9 are indistinguishable

$$\Rightarrow \#\text{ways} = \left(\frac{15!}{(15-9)!}\right)\left(\frac{1}{9!}\right)$$

$$= {}_nC_r$$