

STATISTICS FOR ENGINEERS ASSIGNMENT 14 ANSWERS

This homework assignment primarily deals with Chapters 8 and 9. The problems are worth 2, 2, 3, and 3. You would be silly not to use the computer for this assignment.

Make sure you show your work.

PROBLEM 14.1:

I would like to determine if the unemployment in the United States significantly differed from region to region. To answer this question, I decided to use a one-way analysis of variance procedure. In R, the function call I used was

```
summary(aov(unemp90 ~ region) )
```

As this is from the `crime` dataset, the `unemp90` variable is the unemployment rate in the state in 1990. The `region` variable is a categorical variable of the region in which the state is located. The following is the ANOVA table as displayed by R. Use it to answer the subsequent questions. In each answer you give, make sure you cite the correct information from the table, as well as answer using a complete thought.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
region	8	18.655	2.3319	2.1296	0.05403	.
Residuals	42	45.991	1.0950			

Questions:

- (a) How many regions are there?
- (b) How many data points are there?
- (c) Does the unemployment rate significantly vary from region to region?

Solution:

- (a) The number of degrees of freedom on the `region` variable in the ANOVA table is one less than the number of regions. Thus, there are 9 regions. In fact, the `region` variable is just a renamed `census9` variable.
- (b) The total number of degrees of freedom is one less than the total number of data points. Thus, there are 51 data points. Incidentally, this is just the 50 states plus the District of Columbia.
- (c) As the unemployment rate is the dependent variable, we need only look at the p-value reported and compare it to our usual α value. Here, the p-value is 0.05403, which is larger than $\alpha = 0.05$. Thus we cannot reject the null hypothesis and we must conclude that this data supports the contention that the unemployment rate does *not* vary from region to region.

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PROBLEM 14.2:

Those who study statistical mechanics, thermodynamics, or chemistry define the total energy of a thermodynamic system as enthalpy. Thus, the units of enthalpy are the same as the units for energy, and its value is calculated by determining the energy flows in the system or experiment.

Let us conduct an experiment where we measure the enthalpy for table salt (NaCl) under a variety of methane concentrations. The table below gives the data. Note that there are four concentrations and there are four repeated measurements for each concentration.

Concentration (%)	Enthalpy			
5	1.62	1.60	1.62	1.66
10	2.69	2.66	2.72	2.73
15	3.56	3.45	3.65	3.52
20	3.35	3.18	3.40	3.06

Questions:

- (a) Statistically speaking, is the enthalpy the same at all concentrations? Explain.
- (b) Which pairs of concentrations, if any, can you conclude to have differing enthalpies?

Solution:

- (a) To answer this question, we will have to compare the means of the four concentration groups. To do this, we use one-way ANOVA. Using R, we get the following ANOVA table:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
concentration	3	8.5768	2.85894	345.58	6.55e-12	***
Residuals	12	0.0993	0.00827			

Thus, since the p-value is less than our typical $\alpha = 0.05$, we can reject the null hypothesis that the enthalpies at each concentration are the same and conclude that they are different.

- (b) To answer this question, we need to conduct pairwise means tests. The six t-tests indicate that only 15% and 20% concentrations are not statistically different. Of course, we need to adjust our p-values because we are performing multiple tests. I chose the Bonferroni, since it is easily performed. As there are $n = 6$ tests, we multiply each p-value by 6 and compare *that* value to our usual $\alpha = 0.05$. Alternatively, we can use Tukey's HSD or one of many other adjustments. Using the `TukeyHSD()` function in R, we find that *all* of the groups are statistically separate.

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PROBLEM 14.3:

Using the `crime` data, and using the `census4` variable as the grouping (independent) variable and the `vcrime90` variable as the measurement (dependent) variable, calculate the following:

- (1) Grand mean
- (2) $SSTr$
- (3) SSE
- (4) Total sum of squares
- (5) $MSTr$
- (6) MSE
- (7) Total MS
- (8) Degrees of freedom for the treatment
- (9) Degrees of freedom for the error
- (10) Total degrees of freedom
- (11) The test statistic
- (12) The p-value

Now, create a boxplot of `vcrime90` by `census4`.

Solution: For all but the grand mean, I performed ANOVA and got the results from the table provided (or from calculations on the table).

- (a) Grand mean = 571.8137.
- (b) $SSTr = 1,105,176$.
- (c) $SSE = 6,476,614$.
- (d) Total sum of squares = $1,105,176 + 6,476,614 = 7,581,790$.
- (e) $MSTr = 368,393$.
- (f) $MSE = 137,800$.
- (g) Total MS = Total SS / Total degrees of freedom = $7,581,790 \div 51 = 148,662.5$.
- (h) Degrees of freedom for the treatment = 3.

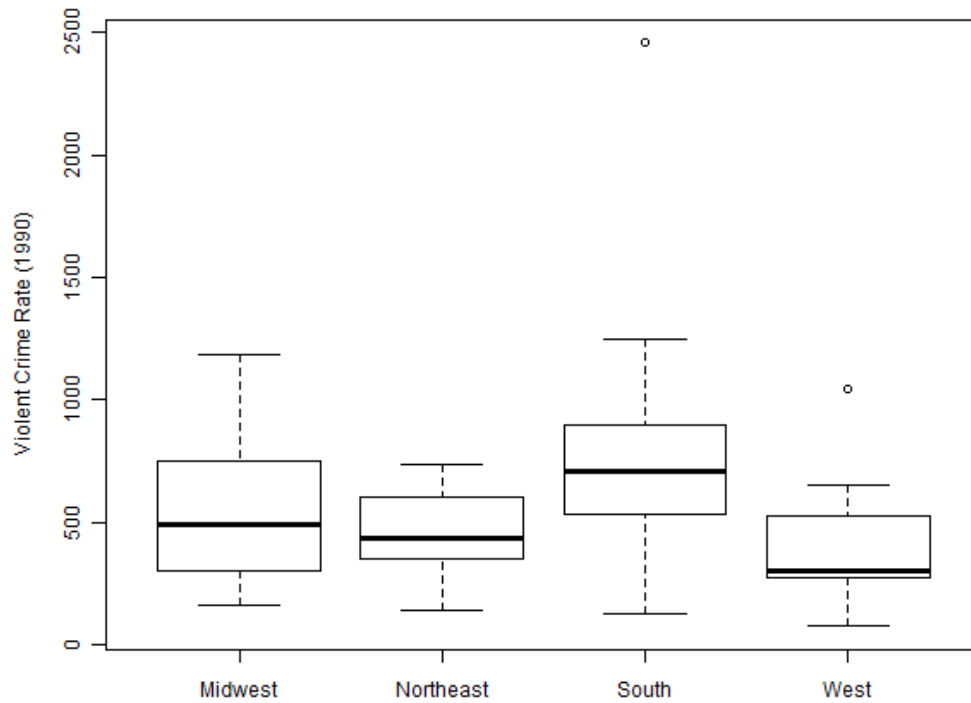


Figure 1. A boxplot of the violent crime rate in 1990 against the region.

- (i) Degrees of freedom for the error = 47.
- (j) Total degrees of freedom = $47 + 3 = 50$.
- (k) The test statistic = 2.6734.
- (l) The p-value = 0.05802.

The boxplot is Figure 1.

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PROBLEM 14.4:

Artificial joints consist of a ceramic ball mounted on a taper. The coefficient of friction determines both the ease of use of the joint and its effective lifetime. A two-factor experiment was performed trying to determine the effects of the taper material and the neck length on the coefficient of friction for the artificial joint. The resulting data is in the following table.

Taper Material	Neck Length	Coefficient of Friction				
CPTi-ZrO ₂	Short	0.254	0.195	0.281	0.289	0.220
CPTi-ZrO ₂	Medium	0.196	0.220	0.185	0.259	0.197
CPTi-ZrO ₂	Long	0.329	0.481	0.320	0.296	0.178
TiAlloy-ZrO ₂	Short	0.150	0.118	0.158	0.175	0.131
TiAlloy-ZrO ₂	Medium	0.180	0.184	0.154	0.156	0.177
TiAlloy-ZrO ₂	Long	0.178	0.198	0.201	0.199	0.210

Questions:

- (a) Compute the main effects and interactions.
- (b) Construct the ANOVA table.
- (c) Is the additive model plausible (in a statistical sense)? Provide the value of the test-statistic and its p-value.
- (d) Can the effect of taper material on the coefficient of friction be described by interpreting the main effects of the material? If so, do so. If not, explain why not.
- (e) Can the effect of the neck length on the coefficient of friction be described by interpreting the main effects of the neck length? Do so. Make sure you determine which pairs of effects differs. Use Tukey's HSD procedure to do this.

Solution:

- (a) The grand mean of the coefficient of friction variable is 0.2156, the mean for the CP group is 0.2600, the mean for the Alloy group is 0.1713. Thus, the effect of CP is $0.2600 - 0.2156 = 0.0444$. Likewise, the effect of the Alloy is -0.0444 . In tabular form,

CPTi-ZrO2	TiAlloy-ZrO2
0.0444	-0.0444

Similarly, the mean of the short, medium, and long necks are 0.1971, 0.1908, and 0.2590, respectively. Thus, the effects of the three necks are

Short	Medium	Long
-0.0185	-0.0248	0.0434

Finally, calculating the effects of the interactions proceeds as it did for the marginal cases. The table of interaction effects is

	Short	Medium	Long
CPTi-ZrO2	0.0063333	-0.023767	0.017433
TiAlloy-ZrO2	0.0063333	-0.023767	0.017433

- (b) The ANOVA table produced is as follows:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
taper	1	0.059052	0.059052	23.6304	5.913e-05	***
length	2	0.028408	0.014204	5.6840	0.009533	**
taper:length	2	0.009089	0.004544	1.8185	0.183924	
Residuals	24	0.059976	0.002499			

- (c) The additive model is plausible, as the interaction term is not statistically significant at the standard α level.

- (d) Yes. As the additive model is plausible, we need only look at the p-value for that effect. As $p < \alpha$, we conclude that the difference in the coefficient of frictions is statistically different between the two taper materials.
- (e) Yes. Again, as the additive model is plausible, we need only look at the p-value for that effect. Again, $p < \alpha$, so we conclude that there is a statistically significant difference among the three neck lengths. Performing pairwise t-tests and using the Bonferroni correction tells us that all three are statistically equal. Using Tukey's HSD procedure, we get that long is statistically different from the other two, but short and medium are not statistically different.

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