STATISTICS FOR ENGINEERS
REVIEW PROBLEMS ANSWERS

1. Sample Problems

Problem 1: Shakespeare

Shakespeare, Inc., produces fishing gear, including some rather high-end fishing line. Fishing line is categorized according to its 'weight,' where a line's weight is the amount of weight the line can support without breaking. To ensure quality control, five lines were tested for each of two shifts. The data are: Shift 1 = 10, 12, 9, 12, 11; Shift 2 = 11, 9, 10, 11, 12. Was there a change in line weight between these two shifts?

What is the null hypothesis? What are the sample means for each of the two samples? What are the variances for each of the two samples? What is the appropriate test to test the null hypothesis? Does the data support the null hypothesis? What is the p-value?

Solution:

The null hypothesis is that the two populations have the same mean. We can write that as $H_0 : \mu_1 = \mu_2$.

- $\bar{x}_1 = 10.8$; $\bar{x} = 10.6$
- $s_1^2 = 1.7$; $s_2^2 = 1.3$
- The appropriate test is an independent samples t-test without the assumption of equal variances.
- The means are close, so we expect that we will not reject the null. Let us find out. The test statistic is $t = 0.2582$. The degrees of freedom is $\nu = 7.86$. Thus, the p-value is $p = 0.8029$. As such, we do not reject the null hypothesis.
- The p-value is 0.8029.

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Problem 2: Roll for initiative

Your elven paladin is preparing to attack a chaotic evil wizard with his +3 Sword of Damocles. To be able to attack, you need to roll a number greater than 19 on a 20-sided die. You pull out your lucky Orange-Swirlie and hope for the best. You know Orange-Swirlie is lucky, because you tested it once. You rolled it 500 times and it came up a 20 a total of 40 times. Here's the question: Is Orange-Swirlie lucky?

What is the null hypothesis? What is the expected number of 20s in 500 rolls of a fair 20-sided die? Was the number of 20s more than would be normally expected?

Solution:

The null hypothesis is that the probability that the Orange-Swirlie comes up a 20 is $\hat{p} = \frac{1}{20} = 0.05$. Or, we could go with a distributional null hypothesis, $H_0 : \hat{p} \sim \mathcal{N}(p, \frac{p(1-p)}{n})$.

- The expected number of 20s in 500 rolls of a fair d20 is $500 \times 0.05 = 25$.
- To test this, we can use a chi-squared test or we can directly calculate the probability. The chi-squared test has a test statistic of $X^2 = 9.47$. This corresponds to $p = 0.0021$.
- We could have also used the proportions test (page 411) and come up with a similar answer.
- Because $p < \alpha = 0.05$, we can conclude that it is unlikely that the null hypothesis is true. In other words, we reject the null hypothesis in favor of the alternative that the die is not fair.

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Problem 3: AIDS

The United Nation's World Health Organization estimates that the AIDS rate in South Africa is 25%; in Botswana, 33%. To reach those estimates, the WHO surveyed 500 people. The opposition candidate in Botswana claims the WHO estimate is politically motivated and that Botswana has the same AIDS rate as South Africa. Does the data support that contention?

What is the null hypothesis? What is the test you will use to test the hypothesis? What is the test statistic? What is your conclusion with respect to the null hypothesis?

Solution: This question was left intentionally vague. The vagueness gives you some practice in trying to determine the best test and the best answer. This is just a difference in proportions test. The question is, what is your null hypothesis? There are two answers (which will give you equivalent conclusions). I give one of the two answers here.

The null hypothesis is that the AIDS rate in Botswana is 33%.

- I will use a comparison of proportions test.
- The test statistic is $z = \frac{0.33-0.25}{\sqrt{(0.33\times 0.67)/500}} \approx 3.80$.
- The test statistic is so small that we can safely reject the null hypothesis and conclude that it is more likely that the AIDS rate in Botswana is not 25%.

If you decided that your null hypothesis was that the AIDS rate in Botswana was 25%, you will get $z = -4.13$. ◇

Problem 4: Extra-Sensory Perception

Robert Goulet claimed to be a psychic. To test his claims, Dr. Cody Martin performed the following experiment: one of three cards was placed face-down and Goulet had to determine which card it was. This experiment was performed 30 times, and Goulet specified the correct card 22 times. Martin hypothesized that Goulet was guessing.

What is the null hypothesis? If we assume each of the three cards were equally likely (the null hypothesis), what is the probability that Goulet would select the correct card by random guessing? What is the correct test in this situation? What is the appropriate conclusion with respect to the null hypothesis?

Solution:

The null hypothesis is that Goulet was guessing; that is, $H_0 : \hat{p} \sim \mathcal{N}(p = 1/3, \sigma^2 = \frac{0.33(0.67)}{30})$.

- If that Goulet selecting any of the three cards is equally lkely (that he is guessing), then $p = 1/3$.
- The correct test is a comparison of proportions test. Or, maybe it is a chi-squared test. I will use the former.
- Using the proportions test, we get $z \approx 2.68$. Thus, we can reject the null hypothesis that Goulet was guessing.
- How was I able to go from the z-value to the conclusion without any table look-up?

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Problem 5: Niwana

A Baltimore restaurant offers three types of food (Korean, Japanese, and Bar) to three types of students (international, undergraduate, and graduate). The management wishes to determine if there is some relationship between student type and food type ordered. The results are in the table below.

	Korean	Japanese	Bar
International	124	78	4
Undergraduate	34	22	90
Graduate	90	88	115

What is the null hypothesis? What is the appropriate test for this hypothesis? What is the test statistic? What is the correct conclusion with respect to the null hypothesis?

Solution:

The null hypothesis is that there is no relationship between the student classification and the food type. Or, that knowing the student type gives no information about the food ordered. Or either of these two reversed. Or $H_0 : X^2 \sim \chi^2_4$, if you like the distributional hypothesis.

- The appropriate test is the chi-squared test. I cannot think of another appropriate test.
- The expected outcome table is

	Korean	Japanese	Bar
International	79.2	60.0	66.8
Undergraduate	56.1	42.6	47.3
Graduate	112.7	85.4	94.9

Going through the calculations, we find $X^2 = 155.75$ with degrees of freedom $\nu = 4$. This corresponds to a p-value of almost zero.

- Thus, we can conclude that knowing the student type gives us some information about the food type he or she will order (or vice-versa).

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Problem 6: Making faces

Bob has a set of data with the lengths of a watchband manufactured by his company. For quality control reasons, Bob decides his null hypothesis is $H_0 : \mu = 13$. He tests his hypothesis at the traditional $\alpha = 0.05$ level with a z-test, since he knows that the population is Normally distributed and that the population variance is $\sigma^2 = 16$. The test gives him a z-value of $z = 1.65$. Bob decides he wants to calculate the power of this test against the alternative hypothesis $H_A : \mu = 16$.

What is probability of a Type I error? What is the probability of a Type II error? What is the power of this test?

Solution: This problem is good practice in z-tests, since you have to use them forwards and backwards.

Since $\alpha = 0.05$, the probability of a Type I error is 0.05.

- Since the probability of a Type II error depends on the power, let us answer this question last.
- First, let us make an important note. We can calculate that the standard error is $se = \frac{3}{1.65}$. You will definitely want to figure out how we do that. It may come in handy. So, using the formula we derived in class, or the version in the book (Example 6.29), we have

  $$13 + 1.96se = 16 - z_\beta se$$

  As we know the standard error, we solve this for z_β and get

  $$z_\beta = \frac{3}{se} - 1.96 = 1.65 - 1.96 = -0.31$$

  With this, a quick trip to the z-table gives us a (one-tailed) probability of $\beta = 0.62$. This is our power.
- Now, we can go back and answer the second question. As the power of a test is 1 - Type II error rate, we know the Type II error rate is 0.38.

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