STATISTICS FOR ENGINEERS
REVIEW PROBLEMS
(FIXED)

This practice homework set serves as a combination of how I ask questions on the examinations and as practice for the second part of the course. Remember that on tests, I will give all important formulae (I will *not* tell you what the formulae are for, nor will I provide appropriate limits of summation or integration, nor will I supply every formula, however). As such, the test will require you to think about which formula is appropriate and which values you need for that formula. By necessity, homework problems (such as these) are more difficult than examination problems. So, if you have no trouble with these problems, you will have no trouble on the examination.

Again, since this is practice, you are not required to do these problems.

These formulae will be on the examination as they are on this page. It is up to you to know what each means and when to use each. Not all formulae are provided. I omitted those that are simple uses of elementary facts (linearity of expected value, e.g.).

1. Distributions

The probability mass functions (or probability density functions) are provided on their support. Outside the support, the probability is zero. I will not provide the support for the distributions. You should also be able to prove the expected value and variance formulae for all of the probability mass functions. For all of these, note that $q = 1 - p$.

Distribution	pmf (or pdf)	Expected Value	Variance
$K \sim \mathcal{U}(a,b)$	$f(k;a,b) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2+1}{12}$
$K \sim Bern(p)$	$f(k;p) = p$	p	pq
$K \sim Bin(n,p)$	$f(k;n,p) = \binom{n}{k} p^k q^{n-k}$	np	npq
$K \sim Geo(p)$	$f(k;p) = pq^{k-1}$	$\frac{1}{p}$	$\frac{q}{p^2}$
$K \sim \mathcal{P}(\lambda)$	$f(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!}$	λ	λ
$X \sim \mathcal{U}(a,b)$	$f(x;a,b) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$X \sim \mathcal{N}(\mu,\sigma^2)$	$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]$	μ	σ^2
$X \sim Exp(\lambda)$	$f(x;\lambda) = \lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

2. Other formulae

$\overline{X}; \bar{x}; \mu; \frac{\sum x_i}{n}; \sum x_i f(x_i); \int x f(x)\ \mathrm{d}x$

$s^2; \sigma^2; \frac{\sum (x_i - \bar{x})^2}{n-1}; \sum x_i^2 f(x_i) - \mathbb{E}\left[X\right]^2; \int x^2 f(x)\ \mathrm{d}x - \mathbb{E}\left[X\right]^2; \frac{p(1-p)}{n}$

$\mathbb{P}\left[A|B\right] = \frac{\mathbb{P}\left[A \cap B\right]}{\mathbb{P}\left[B\right]}$

${}_nP_r = \frac{n!}{(n-r)!}$

${}_nC_r = \frac{n!}{r!(n-r)!}$

$\sigma^2_{c_1 X_1 + c_2 X_2 + \cdots + c_n X_n} = c_1^2 \sigma^2_{X_1} + c_2^2 \sigma^2_{X_2} + \cdots c_n^2 \sigma^2_{X_n}$

$\sigma_U \sim \left|\frac{dU}{dX}\right| \sigma_X$

$\overline{X} \dot{\sim} \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$

$S_n \dot{\sim} \mathcal{N}(n\mu, n\sigma^2)$

$z = \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}}; t = \frac{\bar{x} - \mu_0}{\sqrt{s^2/n}}$

$\chi^2_\nu = \frac{(O-E)^2}{E}$

3. Sample Problems

Problem 1: Shakespeare

Shakespeare, Inc., produces fishing gear, including some rather high-end fishing line. Fishing line is categorized according to its 'weight,' where a line's weight is the amount of weight the line can support without breaking. To ensure quality control, five lines were tested for each of two shifts. The data are: Shift 1 = 10, 12, 9, 12, 11; Shift 2 = 11, 9, 10, 11, 12. Was there a change in line weight between these two shifts?

What is the null hypothesis? What are the sample means for each of the two samples? What are the varainces for each of the two samples? What is the appropriate test to test the null hypothesis? Does the data support the null hypothesis? What is the p-value?

Problem 2: Roll for initiative

Your elven paladin is preparing to attack a chaotic evil wizard with his +3 Sword of Damocles. To be able to attack, you need to roll a number greater than 19 on a 20-sided die. You pull out your lucky Orange-Swirlie and hope for the best. You know Orange-Swirlie is lucky, because you tested it once. You rolled it 500 times and it came up a 20 a total of 40 times. Here's the question: Is Orange-Swirlie lucky?

What is the null hypothesis? What is the expected number of 20's in 500 rolls of a fair 20-sided die? Was the number of 20s more than would be normally expected?

Problem 3: AIDS

The United Nation's World Health Organization estimates that the AIDS rate in South Africa is 25%; in Botswana, 33%. To reach those estimates, the WHO surveyed 500 people. The opposition candidate in Botswana claims the WHO estimate is politically motivated and that Botswana has the same AIDS rate as South Africa. Does the data support that contention?

What is the null hypothesis? What is the test you will use to test the hypothesis? What is the test statistic? What is your conclusion with respect to the null hypothesis?

There is a definite vagueness to this question. It is deliberate. Let us see what you do.

Problem 4: Extra-Sensory Perception

Robert Goulet claimed to be a psychic.[1] To test his claims, Dr. Cody Martin performed the following experiment: one of three cards was placed face-down and Goulet had to determine which card it was. This experiment was performed 30 times, and Goulet specified the correct card 22 times. Martin hypothesized that Goulet was guessing.

What is the null hypothesis? If we assume each of the three cards were equally likely (the null hypothesis), what is the probability that Goulet would select the correct card by random guessing? What is the correct test in this situation? What is the appropriate conclusion with respect to the null hypothesis?

[1] Alright, this is not true. Robert Goulet was an awesome singer.

Problem 5: Niwana

A Baltimore restaurant offers three types of food (Korean, Japanese, and Bar) to three types of students (international, undergraduate, and graduate). The management wishes to determine if there is some relationship between student type and food type ordered. The results are in the table below.

	Korean	Japanese	Bar
International	124	78	4
Undergraduate	34	22	90
Graduate	90	88	115

What is the null hypothesis? What is the appropriate test for this hypothesis? What is the test statistic? What is the correct conclusion with respect to the null hypothesis?

Problem 6: Making faces

Bob has a set of data with the lengths of a watchband manufactured by his company. For quality control reasons, Bob decides his null hypothesis is $H_0 : \mu = 13$. He tests his hypothesis at the traditional $\alpha = 0.05$ level with a z-test, since he knows that the population is Normally distributed and that the population variance is $\sigma^2 = 16$. The test gives him a z-value of $z \approx 1.65$. Bob decides he wants to calculate the power of this test against the alternative hypothesis $H_A : \mu = 16$.

What is probability of a Type I error? What is the probability of a Type II error? What is the power of this test?

You do not need this information, but $n = 5$. In fact, using this information may make things a bit more difficult for you. But, good luck!

By the way, there will be no power calculations on this test. This is here for other types of practice.