**STATISTICS FOR ENGINEERS**
**ASSIGNMENT IX ANSWERS**
**OCTOBER 29, 2010**

## Problem 9.1

You are in charge of quality control at a barbed wire manufacturing plant. Your plant receives steel wire from the Lindsey Corporation. One week, a sample of 125 lengths of wire had mean breaking strength 6.1 N, with a standard deviation of 0.7N. You do not like these numbers, so you decide to go with a different wire supplier. The next week, you receive a batch of wire from the Navidi Corporation. In a sample of 75 lengths of wire from the new vendor, the mean breaking strength was 5.8N and the standard deviation was 1.0N. Find a 90% confidence interval for the difference in mean breaking strength between the wires supplied by the two vendors.

*Solution:* This is a difference of independent means problem. As such, we use Formula 5.16:

$$\overline{X} - \overline{Y} \pm z_{\alpha/2}\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$$

$$6.1 - 5.8 \pm 1.645\sqrt{\frac{0.7^2}{125} + \frac{1^2}{75}}$$

Thus, the confidence interval for the difference in means is $\mu_X - \mu_Y = (0.084, 0.516)$. (Note that $\alpha = 0.10$ here. Why?)

Thus was not asked, but as zero is not contained in the interval, we know that there is a difference in the mean breaking strengths in the two samples (at the $\alpha = 0.10$ level). ◇

## Problem 9.2

A method of doping a silicon wafer with gallium arsenide is supposed to produce a coating whose mean thickness is no greater than 7 microns ($\mu$m). You are told by a line operator that the doping machine is malfunctioning, so you measure the thickness of 36 coated specimens and test the hypothesis $H_0 : \mu \leq 7$ against $H_A : \mu > 7$. From your tests, you obtain a p-value of 0.40. Since $p > 0.05$, you naturally conclude that the mean thickness is within specification. Is this conclusion correct? Explain.

*Solution:* This conclusion is not correct, although subtly so. You cannot conclude that $H_0$ is correct. You can only conclude that there is not enough evidence to reject $H_0$ in favor of $H_A$.

With that said, of course in the real world, not rejecting has the same result on the line as accepting. But—technically—there is a difference. ◇

## Problem 9.3

Bakugan Industries asserts that their top-line watches have a greater than 95% probability that its readings are within 0.1s of the true time. You decide to test this claim. In a sample of 500 watches, you find that 470 are within 0.1s of the true time. Is there enough evidence to reject the claim? Explain.

*Solution:* At first glance, it would seem that all you would have to do is calculate 470/500-0.94 and conclude that there is not enough evidence to reject the claim. However, this method is incorrect (although it does usually give a good back-of-the-envelope answer). The problem is that this is actually a series of 500 Bernoulli tests. Thus, we use the test for population proportion, based on the assumption that $\hat{p} \sim \mathcal{N}(p, \frac{p(1-p)}{n})$. So, the z-value is

$$z = \frac{470/500 - 0.95}{\sqrt{0.95(0.05)/500}} = -1.026$$

Thus, looking in the z-table, we get a p-value of about 0.1525.

This is a one-tailed test. Why? You are only caring if the number of defective watches is too high. Remember that Bakugan said $H_0 : p \geq 0.95$. Thus, the alternative hypothesis is one-tailed (also known as *directional*). ◇

## Problem 9.4

A voltmeter is used to make five measures of the carbon content of a specific steel beam in a bridge on each of two successive days. The results are as follows:

| | | | | | |
|---|---|---|---|---|---|
| Day 1: | 2.1321 | 2.1385 | 2.0985 | 2.0941 | 2.0680 |
| Day 2: | 2.0853 | 2.1476 | 2.0733 | 2.1194 | 2.0717 |

Can you conclude that the calibration of the voltmeter has changed from the first day to the second? Explain.

*Solution:* There is only one steel beam we are testing. The ten measurements are taken over two days, and it is the *measurements* that we are checking. Thus, we use a difference of independent means test. Here, a t-test, as $n < 30$ (Formula 6.5). To perform such a test, we need to calculate $\mu_1$, $\sigma_1^2$, $\mu_2$, and $\sigma_2^2$ and substitute them into the appropriate formula.

Doing this, we get $t = 0.3444$, with $\nu \approx 7.871$. This gives $p \approx 0.75$. Thus, we cannot reject the null hypothesis with this data. ◇

## Problem 9.5

The crystal thickness of six watch faces was measured to be 9.3, 0.9, 9.0, 21.7, 11.5, and 13.9. For this problem, the measured values definitely do not come from an normal distribution but I am willing to assume symmetry. Test the hypothesis $H_0 : \mu = 20$ against $H_A : \mu \neq 20$.

*Solution:* Since the null hypothesis is of the form $H_0 : \mu = \mu_0$, this is a two-tailed test. The observed value of the test statistic is $S_+ = 1$, with $n = 6$. Table A.4 tells us that the area under the curve to the left of 1 is 0.0.0312. As this is a two-tailed test, we have $p = 0.0624$. As such, we do not have enough evidence to reject the null hypothesis. ◇