STATISTICS FOR ENGINEERS
REVIEW PROBLEMS

This practice homework set serves as a combination of how I ask questions on the examinations and as practice for the first part of the course. Remember that on tests, I will give all important formulae (I will *not* tell you what the formulae are for, nor will I provide appropriate limits of summation or integration, nor will I supply every formula, however). As such, the test will require you to think about which formula is appropriate and which values you need for that formula. By necessity, homework problems (such as these) are more difficult than examination problems. So, if you have no trouble with these problems, you will have no trouble on the examination, which is scheduled for October 1, 2010. The solutions to this practice assignment will be posted on Tuesday, September 21, 2010, which is before the review session scheduled for September 28, 2010.

Again, since this is practice, you are not required to do these problems.

Do not expect the actual examination to be this long. It will consist of five (5) questions (with multiple parts). You will be allowed a scientific calculator for the sole purpose of using the basic functions: `(`, `+`, `-`, `*`, `/`, `^`, and `)`.

1. Formulae

These formulae will be on the examination as they are on this page. It is up to you to know what each means and when to use each. Not all formulae are provided. I omitted those that are simple uses of elementary facts (linearity of expected value, e.g.).

$$\overline{X}; \bar{x}; \mu; \frac{\sum x_i}{n}; \sum x_i f(x_i); \int x f(x)\ \mathrm{d}x$$

$$s^2; \sigma^2; \frac{\sum (x_i - \bar{x})^2}{n-1}; \sum x_i^2 f(x_i) - \mathbb{E}\left[X\right]^2; \int x^2 f(x)\ \mathrm{d}x - \mathbb{E}\left[X\right]^2$$

$$\mathbb{P}\left[A|B\right] = \frac{\mathbb{P}\left[A \cap B\right]}{\mathbb{P}\left[B\right]}$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

$$\sigma^2_{c_1X_1+c_2X_2+\cdots+c_nX_n} = c_1^2\sigma^2_{X_1} + c_2^2\sigma^2_{X_2} + \cdots c_n^2\sigma^2_{X_n}$$

$$\sigma_U \sim \left|\frac{dU}{dX}\right| \sigma_X$$

$$\overline{X} \dot{\sim} \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

$$S_n \dot{\sim} \mathcal{N}(n\mu, n\sigma^2)$$

1.1. **Distributions.** The probability mass functions (or probability density functions) are provided on their support. Outside the support, the probability is zero. I will not provide the support for the distributions. You should also be able to prove the expected value and variance formulae for all of the probability mass functions. For all of these, note that $q = 1 - p$.

Distribution	pmf (or pdf)	Expected Value	Variance
$K \sim \mathcal{U}(a, b)$	$f(k; a, b) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2+1}{12}$
$K \sim Bern(p)$	$f(k; p) = p$	p	pq
$K \sim Bin(n, p)$	$f(k; n, p) = \binom{n}{k} p^k q^{n-k}$	np	npq
$K \sim Geo(p)$	$f(k; p) = pq^{k-1}$	$\frac{1}{p}$	$\frac{q}{p^2}$
$K \sim \mathcal{P}(\lambda)$	$f(k; \lambda) = \frac{e^{-\lambda}\lambda^k}{k!}$	λ	λ
$X \sim \mathcal{U}(a, b)$	$f(x; a, b) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$X \sim \mathcal{N}(\mu, \sigma^2)$	$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]$	μ	σ^2
$X \sim Exp(\lambda)$	$f(x; \lambda) = \lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

2. Sample Problems

Problem This is a quiz

A quiz is given to the class. It consists of 5 True-False questions and 5 multiple-choice question, with four options each. How many ways can the quiz be answered? Assuming that a certain student guesses on all of the questions, what is the probability that the student will Ace the quiz (get 9 or 10 correct)?

Problem A license to drive

The license plates for the great state of Oregon consists of six spaces. The first space is letter that signifies the month the plate expires. Possible values are A, B, C, D, E, F, G, H, J, K, L, and M. The next two spaces consist of letters other than O and I. The final three spaces consist of digits (zero through nine). How many license plates can Oregon produce under this scheme? If every license plate is created, what is the probability that I will be issued `MER 101` as my plate? What is the probability that I will be issued either `OLE 007` or `OLE 666` as my plate?

Problem The blackbird

In general, let us denote having the disease by D, not having the disease by $\sim D$, testing positive by $+$ and testing negative by $-$.

Doctor researchers have just completed trials on a new clinical test, called SR-71. They concluded that the false-positive rate is $\mathbb{P}\left[+|\sim D\right] = 0.100$, and the false-negative rate is $\mathbb{P}\left[-|D\right] = 0.010$. The prevalence of the disease in society is $\mathbb{P}\left[D\right] = 0.0015$. What is the true-negative rate ($\mathbb{P}\left[-|\sim D\right]$)? What is the true-positive rate ($\mathbb{P}\left[+|D\right]$)?

Now, using Bayes' Rule (page 80), what is the probability of having the disease, given that the person tested positive ($\mathbb{P}\left[D|+\right]$)? What is the probability of having the disease, given that the person tested negative ($\mathbb{P}\left[D|-\right]$)?

Problem Declaration of dependence

Molecular Biophysicists conduct the following experiment. Light from one of three lasers (100Å, 4000Å, and 7000Å) is shown on a DNA base pair, and the event of disintegration of the base pair is measured. A laser is categorized according to its light output. The results of the experiments are provided in the following table.

	100Å	4000Å	7000Å
Disintegrations	2	6	3
Non-disintegrations	8	24	12

Are the effects of the lasers independent of the disintegration of the DNA base pairs? (**Note**: This is the opposite question as asking if the lasers affected the disintegrations differently.)

Problem Risk it all

Rolling a fair die is a Bernoulli trial, depending on how we determine the definition of success. In the game of Risk, the winner of a battle is determined by rolling dice. If the challenger rolls *higher* numbers, he or she wins the battle. Otherwise, the challenger loses. Julie challenges Tom to a battle. Julie decides to roll one die; Tom, also. What is the probability that Julie wins the battle?

Problem Under a little pressure

Let us suppose that air enters a compressor at pressure $P_1 = 10.1 \pm 0.3$ MPa and leaves the compressor at $P_2 = 20.1 \pm 0.3$ MPa. From our knowledge of fluids, we know that the intermediate pressure in the line is the geometric mean, $P_3 = \sqrt{P_1 P_2}$. Find the intermediate pressure, including the appropriate uncertainty. Which would provide a greater reduction in uncertainty in your estimate of P_3: reducing the uncertainty in your measurement of P_1 to 0.2 MPa or reducing the uncertainty in your measurement of P_2 to 0.2 MPa?

Problem Einsteinium

A radioactive isotope of Einsteinium, ^{253}Es, emits an alpha particle from time to time according an exponential distribution with rate parameter 0.049 day^{-1}. What is the probability density function (pdf) for this isotope? What is its cumulative distribution function (cdf)? What is the expected time between emissions? What is the variance of the time between emissions? What is the probability that this isotope will *not* emit an alpha particle in the next 40 days? What is the probability that this isotope will not emit an alpha particle in the next 40 days *given that it has not released an alpha particle in the last 120 days*?

Problem Fits you to a T (cell)

In a medical test, Human T-Cells are suspended in a medium. After 145 minutes, samples are taken from the medium, and the life-status of the T-Cells, which have not increased in size or number, is determined. The researcher originally placed 1.50×10^6 T-Cells in 1.510L of media. She then stirred the medium to achieve uniform density of T-Cells in the medium. At the end of the experiment, 145 minutes later, she removed 1.000mL of medium. How many T-Cells would we expect her to have withdrawn along with the medium? What is the probability that she removed none?

Problem More positioning tubes

In a certain production process, the lengths of manufactured positioning tubes are independently distributed normally with mean 12.10cm and standard deviation 1.00cm. In one shift, 5000 are produced. You select one at random and measure its length. What is the probability that it measure less than 11.99cm? You select another at random from the remaining 4999. Given that the first one measured 12.00cm, what is the probability that this second tube measures less than 11.99cm in length?

Problem Take a chance?

A certain professor makes a deal with his students on an upcoming True-False examination. If any student answers all 20 True-False questions *incorrectly*, then the student receives a score of 150% on the examination (30/20). Otherwise, the student receives the number correct out of 20. Let us assume that a certain student's knowledge about Question i is independent of his or her knowledge about Question j. Under this assumption, we are looking at a series of 20 independent Bernoulli trials, each with probability of success being p. Student Abel believes he has a 50% chance of answering all of the questions correctly. What chance does he have of answering a single question correctly? Student Bourbaki thinks he has a 99.5% of answering each question correctly, what is his probability of answering all 20 questions correctly?

Problem Missed the bus

Let us assume that bus arrival time is exponentially distributed, with a rate parameter of 0.05 (min^{-1}). You arrive at the bus stop, just seeing a bus leaving as you approach the stop. How long should you expect to wait for the next bus? Let us make this problem a bit more interesting. Let us suppose that there are five bus lines that service your stop. All of the busses look identical from the rear. Thus, you do not know which bus you just missed. Now, how long should you expect to wait for *your* bus?

Problem Short-lived connections

The Exponential distribution is often used in survival time analysis. It has one parameter, λ. It has been discovered that the connector component of a circuit board has a lifetime in minutes, T, that is distributed as $T \sim Exp(\lambda = 0.01)$. How long should we expect the connector to last?

Problem A shot in the dark

A flashlight requires two functional batteries to operate correctly. You go spelunking in the mountains of southwestern Oklahoma searching for peace and relaxation. Spelunking requires a flashlight. You pack four (4) batteries with your flashlight, thinking that will be sufficient for your exploration. The lifetime (in minutes) of these batteries are independently distributed $T \sim Exp(\lambda = 0.01)$. Assuming you are able to change out dead batteries instantly, how long will your flashlight work?