4 Monte Carlo Methods

Note Title 9/13/2011

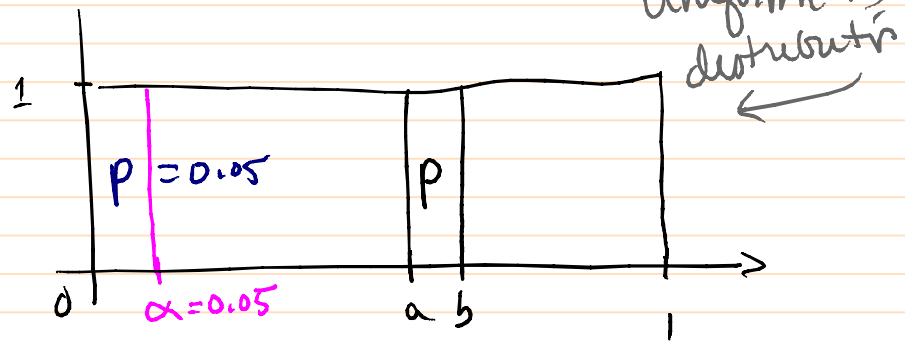
Today: Monte Carlo experiments
- testing a test — §2.2.1 p 37
- estimating a distribution for a test statistic — §A.1.5 p 325

future: - Answering "probability of an event" questions — §4.4.5 p 130

1 Testing a test

Fact: pvalues are uniformly distributed if the test is appropriate.

Thus, a histogram of the pvalues should look like this →



example: Testing the t-test. 1-sample

Assumption: Normal distributed measurements

CORE

$H_0 \rightarrow X \sim N(\mu = 0, \sigma^2 = 1)$

test $\rightarrow$ t.test(X, mu = 0)

overhead

```
p <- numeric( )
n <- 60
set.seed (18)
```

Loop

```
for(i in 1:10000) {
```

core

```
    X <- rnorm(n ,m=0 , s=1 )

  p[i]<- t.test (x , mu=0 )$p.value
```

```
}
```

← braces enclose the core

#2. To create a test (A.1.5)

small sample size?

and not Normally distributed?

and you know how your data is distributed!

If these are true, then you should do this.

Rarely will you know your data's distribution. When you do, it will likely be Binomial.

from the data

$$X_A \sim Bin(n = 10, \pi = 4/10)$$

$$X_M \sim Bin(n = 10, \pi = 2/10)$$

$$H_0: \quad \pi_A = \pi_M \implies \pi_A - \pi_M = 0$$

$$\bar{X}_A - \bar{X}_M = TS$$

Step 1. Devise a test statistic.

Requirements:

- Values of TS close to zero correspond to a true H_0.
- Values of TS far from zero ... false H_0.

Step 2. Determine distribution of TS

- estimate it using Monte Carlo

$0.3 = \frac{0.4 + 0.2}{2}$

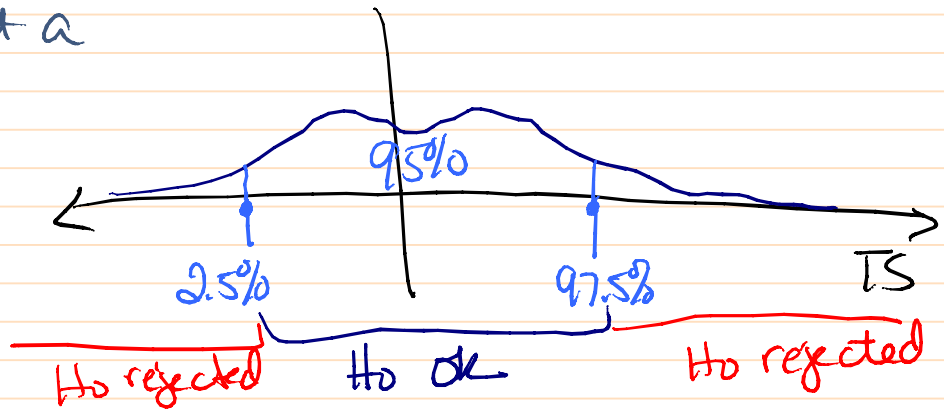
Core

```
xa <- rbinom(1, 10, 0.3)
xm <- rbinom(1, 10, 0.3)
TS[i] <- xa - xm
```

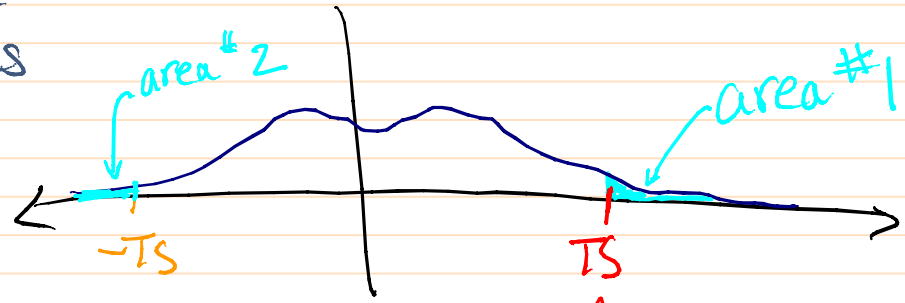
Step 3. Calculate the confidence interval and/or the p-value.

Let us pretend this ↓ is the dist of our TS

Schematic of what a 95% confidence interval means:



Schematic of what a p-value means



Observed test statistic

p-value = Area #1 + Area #2

= probability of observing a TS more extreme than you observed.

In R:

95% confidence interval:

```
quantile(TS, c(0.025, 0.975))
```

p-value:

```
length(which(TS > TS0))/trials * 2
```

TS0: Observed test statistic

2: two-tailed test